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Xinzhuang Cui,[2] Jiong Zhang,[1] Fei Hou,[2] Zhijun Gao,[2] Zengtang Wang,[3] and Wei Sui[2]

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[1] School of Civil Engineering, Shandong University, 17922 Jingshi Road, Jinan, Shandong 250061, China (Corresponding author).

[2] School of Civil Engineering, Shandong University, 17922 Jingshi Road, Jinan, Shandong 250061, China.

[3] Weifang Highway Bureau of Luqiao Construction, Engineering Development Center, Weifang, Shandong 261031, People's Republic of China, China.

ABSTRACT

When the height of roller compacted concrete (RCC) specimen is less than 400 mm, the specimen size and crack depth ratio will affect the double-K fracture parameters. In this paper, two groups of RCC specimens with different specimen size and crack depth ratio were investigated and the wedge splitting method was employed. Double-K fracture parameters were calculated employing the double-K fracture theory of concrete. It is indicated that when the crack depth ratio, α, increased from 0.4 to 0.6, the unstable fracture toughness, K_{IC}^{un}, increases, while the initial fracture toughness, K_{IC}^{ini}, decreases. The crack depth ratio, α, has a continuous effect on K_{IC}^{ini}, however the effect on K_{IC}^{un} decreases with the increase of α. When the height of the specimen, *h,* is less than 300 mm, the effect of specimen size on K_{IC}^{un} is significant, but it decreases when *h* is greater than 300 mm and less than 500 mm.

Keywords

wedge splitting test, specimen size, crack depth ratios, double-K fracture parameters

Introduction

Unlike conventional concrete (CC), roller compacted concrete (RCC) is a drier mix, stiff enough to be compacted with vibratory rollers. Typically, RCC is constructed without joints and needs neither forms nor finishing, nor does it contain dowels or steel reinforcing. These characteristics make RCC a simple, fast, and economical mix, and as such, more and more countries have begun to use RCC as an effective pavement solution [1]. Due to its advantages as a comparatively low cost, durable paving material, RCC has been used widely in urban street reconstruction, residential subdivision roads, and some heavy traffic pavements. However, like CC, cracking is a serious problem for RCC.

Traditional strength theory provides a criterion for judging the failure of concrete. Shihata [2] conducted an experimental program to study the effect of compaction on the strength of RCC specimens. However, the traditional theory ignores the effect of specimen size on load-carrying capacity. By analyzing the stress and strain fields at the crack tip, fracture mechanisms introduced the concept of fracture toughness to reflect the ability to resist crack propagation in concrete materials [3]. It is used widely to analyze the failure of concrete structure because it can be employed to estimate the response of structure more accurately by reasonably explaining the size effect. Ramsamooj [4] presented an approximate analytical model for the fatigue crack propagation rate in concrete; the model considers fatigue cracking at all stress levels up to the modulus of rupture. Nguyen et al. [5] studied various fatigue criteria with indirect tensile mode of loading. A new approach was proposed to identify the fatigue failure based on a crack-length criterion. In plastic limit analysis or elasticity with strength limit, the nominal strength of concrete is size independent; this phenomenon is called the size effect. Bazant and Kazemi [6] proposed the size effect model and studied the fracture and size effect in concrete [7]. Several studies have been done on crack development and fracture mechanisms of the RCC. Nallathambi and Karihaloo [8] studied the effects of the specimen and crack sizes upon fracture toughness of concrete. Pittman and McCullough [9] carried out experiments to study the crack development of RCC pavement. Based on toughness and fracture tests, Albuquerque et al. [10] studied the fracture characterization of RCC and determined the crack extension resistance curve of the concrete mixtures. In order to avoid the non-ordered cracks, induced cracks are adopted in the RCC. Based on the characteristics of the induced cracks, Liu and Song [11] employed a fictitious crack model and linear elastic fracture mechanics to study equivalent strength of induced cracks. Results indicated that equivalent strength could be used as the fracture criterion for induced cracks. Zhang et al. [12] evaluated the fracture parameters and the critical crack length of RCC and provided a calculation method for the initial fracture and development of induced cracks.

Xu and Zhao [13] put forward the double-K (the initial fracture toughness, K_{IC}^{ini}, and the unstable fracture toughness, K_{IC}^{un}) fracture theory and gave a simple and practical crack fracture criterion. The use of three point bending beam test and wedge-splitting test (WST) to determine the double-K fracture parameters of concrete is recommended by the Chinese norm for fracture test of hydraulic concrete [14]. However, the specimen size and crack depth ratio will affect the double-K fracture parameters. Moreover, minimal research has been done in this area. In this paper, WST is employed to study the effect of specimen size and crack depth ratio on double-K fracture parameters.

Double-K Fracture Theory of Concrete

Based on the stress intensity factor of concrete and the concept of virtual cracks that reflect the softening properties of concrete, the double-K fracture theory is a more complete theory used to describe the fracture of concrete. The double-K criterion can predict the different stages of crack propagation during the fracture process in quasi-brittle materials. According to this criterion, the two size independent parameters, K_{IC}^{ini} and K_{IC}^{un}, can be used to describe the fracture process of concrete [15].

Double-K fracture theory of concrete can be described as follows: comparing with K_{IC}^{ini} and K_{IC}^{un}, if the crack tip stress intensity factor, K, is less than K_{IC}^{ini}, the concrete structure has no cracks. When K is greater than K_{IC}^{ini} but less than K_{IC}^{un}, cracks occur and begin to extend steadily. If K is greater than K_{IC}^{un}, the cracks of the structure start to extend unsteadily.

For the concrete structures in any shape, K can be calculated by many numerical methods. In practical application, $K = K_{IC}^{ini}$ is the judgment criterion of the crack propagation in main structure. $K_{IC}^{ini} < K < K_{IC}^{un}$ can be used as the security alert of the unstable propagation of the main structure. $K > K_{IC}^{un}$ is the judgment criterion of the general structure.

Preparation of RCC Specimens

When the height of specimens is greater than 400 mm, the double-K fracture parameters are approximately constant [16]. So in this study, the sizes of the specimens were ~400 mm. The specimens were prepared according to the Chinese Test Methods of Cement and Concrete for Highway Engineering [17]. All the RCC specimens were made by the same batch, which had the same mix proportions of concrete, the materials, and the compressive strength of the RCC specimens as shown in **Table 1**. The cement was P.O32.5R silicate cement, the river sand with grain size <5 mm was used as the fine aggregate, and the coarse aggregate was the crushed stone with grain size <20 mm. All the materials were mixed and fully stirred, then the mixture was

TABLE 1 The materials and the compressive strength of RCC.

Water (%)	Cement (%)	Fly Ash (%)	Fine Aggregates (%)	Coarse Aggregates (%)	Additive (%)	f_{cu} (MPa)
5.33	5.40	3.69	32.35	52.48	0.75	37.87

TABLE 2 The sizes and crack depth ratios of the two groups of specimens.

Group	l by h by t (mm by mm by mm)	α	a (mm)	m (mm)
Group1	150 by 150 by 150	0.4	60	3
	150 by 150 by 150	0.5	75	3
	150 by 150 by 150	0.6	90	3
Group2	300 by 300 by 150	0.4	120	3
	400 by 400 by 150	0.4	160	3
	500 by 500 by 150	0.4	200	3

placed into the molds, and a vibrating hammer was used to produce specimens with two layers. After casting, the specimens were covered with plastic film until the next day when they were de-molded. The specimens were then stored in water, with a temperature of $21 \pm 2°C$, until the time of testing that in most cases took place 28 days after casting.

A total of 18 specimens were prepared. Three types of specimens were included in group 1 and group 2. There were 3 RCC specimens in each type, as shown in **Table 2**. The specimen is equipped with two blocks of marble (for applying the splitting force), and the shape and size of the RCC specimen is shown in **Fig. 1**, and the real RCC specimen is shown in **Fig. 2**. In **Table 2** and **Fig. 1**, l, h, t are the length, height and thickness of the specimens, respectively, α is the crack depth ratio, a is the length of the crack, and m is the thickness of the crack.

Wedge Splitting Test (WST)

In this paper, WST was employed. The WST method, originally proposed by Linsbauer and Tschegg [18] and later developed by Brühwiler and Wittmann [19], is a simple method since it does not require sophisticated test equipment and the result is stable. The WST method has been proved successful for the determination of fracture properties of concrete [20,21].

FIG. 1 Shape and size of RCC specimen.

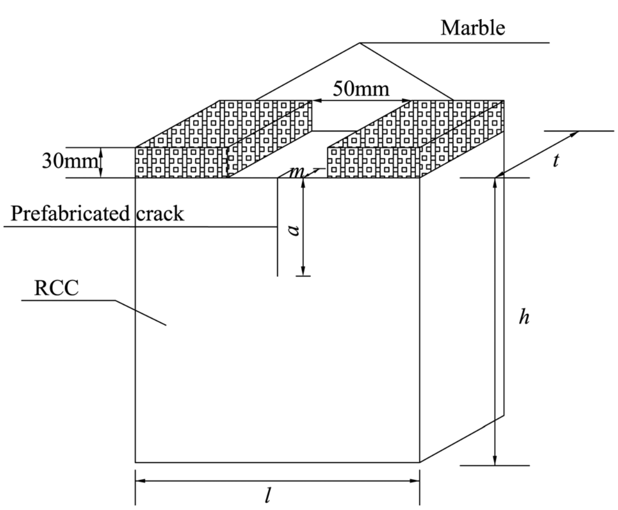
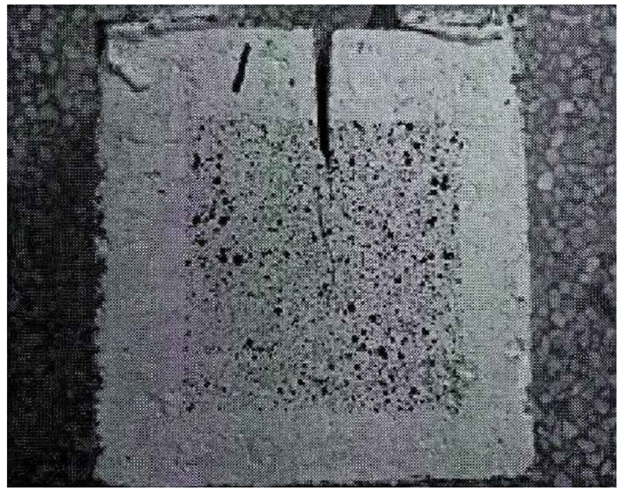


FIG. 2 RCC specimen.



A schematic of the testing equipment is clarified in **Fig. 3**, and the real testing picture shown in **Fig. 4**. Two steel plates with roller bearings and the wedge-shaped frame are placed partly on top of the specimen and partly into the groove that is between the two blocks of marble. On the surface of the upper platen, there is a reservation groove, through which the clip-on extensometer can be installed on the surface of the RCC specimen. During the test, the force on the specimen (F) and the crack mouth opening displacement ($CMOD$) can be measured by the force sensor and the clip-on extensometer, respectively. Through the signal acquisition instrument, the data can be acquired and sent to the computer, then F and $CMOD$ obtained. The typical F-$CMOD$ curve is shown in **Fig. 5**. The $CMOD_c$ is $CMOD$ corresponding to $F_{\max}$.

Calculation Methods of Double-K Fracture Parameters

For specimens with different sizes, the calculation methods of the double-K fracture parameters are different. For the specimen with the size of 150 by 150 by 150 mm, according to the Chinese specification of norm for fracture test of hydraulic concrete (DL/T5332-2005), the double-K fracture parameters can be calculated as follows:

Unstable fracture toughness, K_{IC}^{un}, is determined by Eq 1:

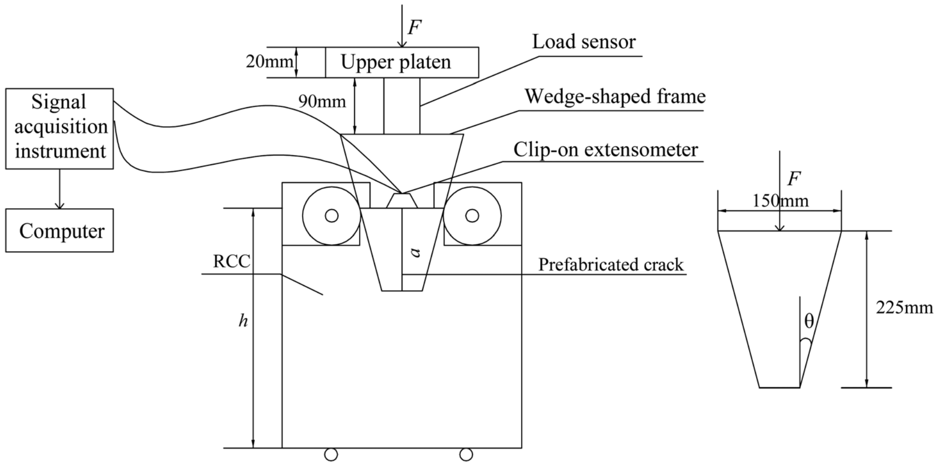
$$K_{IC}^{un} = \frac{F_{H\max} \times 10^{-3}}{h\sqrt{t}} \times f(\alpha) \tag{1}$$

where:

$F_{H\max}$ = the maximum horizontal force, determined by Eq 2,

t and h = the thickness and height of specimen, as shown in **Fig. 1**, $f(\alpha) = (3.675[1 - 0.12(\alpha - 0.45)])/(1 - \alpha)^{3/2}$, $\alpha = (a_c/h)$, and

FIG. 3 Schematic of the equipment.



a_c = effective crack length, determined by Eq 3.

$$F_{H\max} = \frac{F_{\max} + mg \times 10^{-2}}{2\tan\theta} \quad (2)$$

where:

$F_{\max}$ = the maximum force, as shown in **Fig. 5**,

m = the mass of the wedge loading frame,

g = the acceleration due to gravity, and

θ = the angle between wedge loading frame wedge surface and the longitudinal axis, as shown in **Fig. 3**.

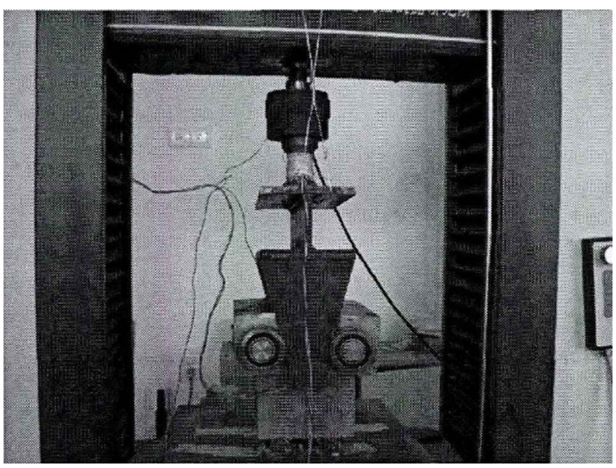
$$a_c = (h + h_0)\left[1 - \left(\frac{13.18F_{H\max}}{tECMOD_c} + 9.16\right)^{1/2}\right] - h_0 \quad (3)$$

where:

h_0 = the thickness of the steel plate in clip-on extensometer,

$CMOD_c$ = the critical crack mouth opening displacement, as shown in **Fig. 5**, and

FIG. 4 Real testing picture.



E = the calculated elastic modulus (GPa), determined by the following equation:

$$E = \frac{1}{tc_i}\left[13.18\left(1 - \frac{a_0 + h_0}{h + h_0}\right)^{-2} - 9.16\right] \quad (4)$$

where:

c_i = the initial value of $CMOD/F$,

$CMOD$ = the crack mouth opening displacement, as shown in **Fig. 5**,

F = the force on the specimen, as shown in **Fig. 3**, and

a_0 = the initial crack length.

Initial fracture toughness K_{IC}^{ini} is determined by Eq 5:

$$K_{IC}^{ini} = \frac{F_{HQ} \times 10^{-3} \times f(\alpha)}{t\sqrt{h}} \quad (5)$$

where F_{HQ} is the crack initiation horizontal load, determined by Eq 6:

$$F_{HQ} = \frac{F_Q + mg \times 10^{-2}}{2\tan\theta} \quad (6)$$

where F_Q is the crack initiation load.

FIG. 5 Typical F-CMOD curve.

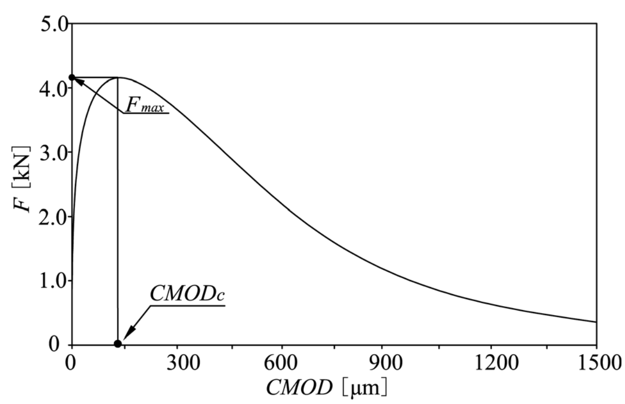
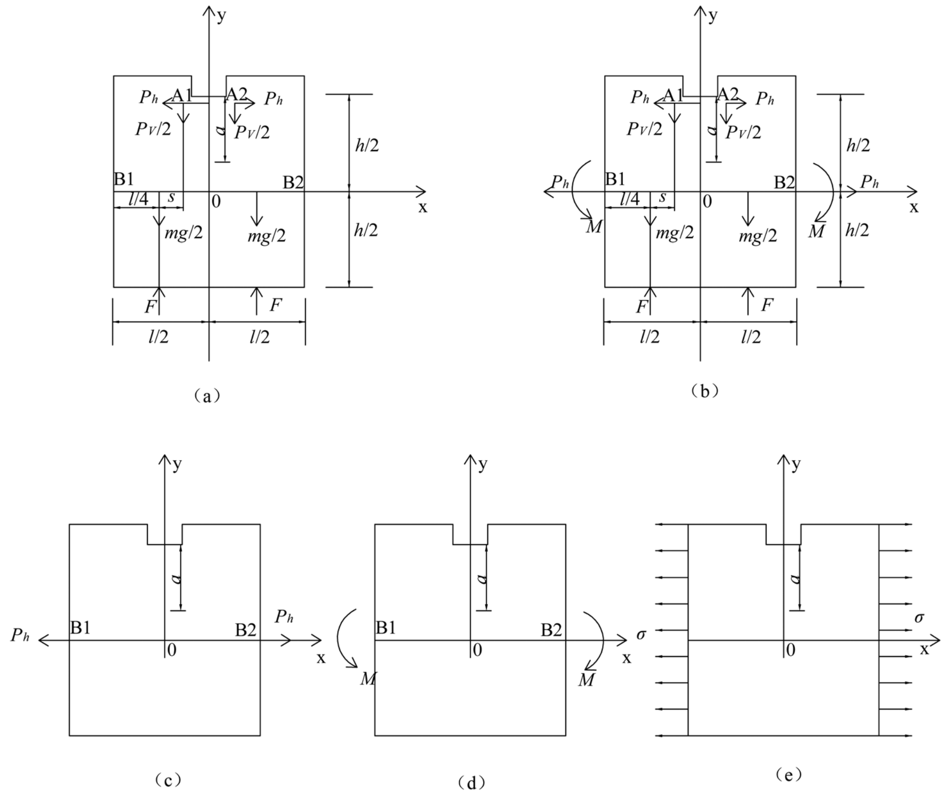


FIG. 6

Force conditions and equivalent force analysis of RCC specimen.



For the specimen with sizes larger than 150 by 150 by 150 mm, the coordinate system with the center of specimen as the origin is drawn as in **Fig. 6(*a*)**, where A1 and A2 is force points of the wedge loading frame; P_h is the horizontal force and P_v is the vertical force from the wedge loading frame; F is the support force. According to the balance of force, $F = \frac{1}{2}mg + \frac{1}{2}F_v$, when the symmetrical force of both side moved to B1 and B2 (as shown in **Fig. 6(*a*)**), the stress system of **Fig. 6(*a*)** is equivalent to **Fig. 6(*b*)**, where M is the equivalent bending moment. According to the superposition principle of intensity factor [22], **Fig. 6(*b*)** can be decomposed into the **Fig. 6(*c*)** and **Fig. 6(*d*)**, and **Fig. 6(*c*)** is equivalent to **Fig. 6(*e*)**. Then the stress condition of the specimen turns into the sum of uniaxial tension and pure bending moment.

So the unstable fracture toughness, K_{IC}^{un}, can be expressed as Eq 7:

$$K_{IC}^{un} = K_1 + K_2 \tag{7}$$

K_1 is the unstable fracture toughness for the specimen under uniaxial tension, determined by Eq 8:

$$K_1 = \sigma(\pi a_c)^{\frac{1}{2}} \times f\left(\frac{a_c}{h}\right) \tag{8}$$

where $f(a_c/h) = 1.122 - 0.231(a_c/h) + 1.055(a_c/h)^2 - 21.71(a_c/h)^3 + 30.382(a_c/h)^4$; $\sigma = P_h/th$.

K_2 is the unstable fracture toughness for the specimen under pure bending moment, determined by Eq 9:

$$K_2 = \frac{6M}{th^2}(\pi a_c)^{\frac{1}{2}} \times g\left(\frac{a_c}{h}\right) \tag{9}$$

where $g(a_c/h) = 1.122 - 1.40(a_c/h) + 7.33(a_c/h)^2 - 13.08(a_c/h)^3 + 14.0(a_c/h)^4$; $M = (1/2)P_h - (1/2)sP_v$.

Results and Analyses

The crack in the RCC specimen during the test is shown in **Fig. 7**.

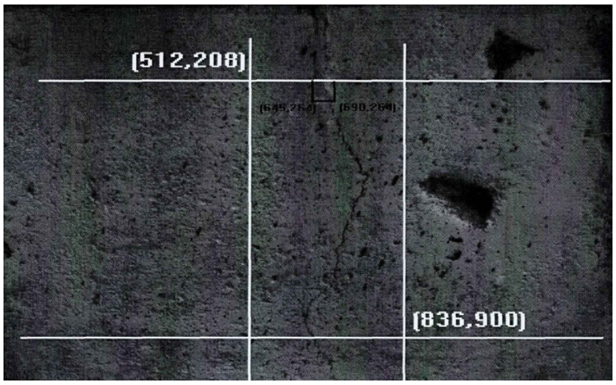
In **Fig. 7**, the subcritical effective crack can be clearly seen; the crack develops roughly. This phenomenon is caused by the aggregate in the RCC specimen, when the crack meets the aggregates, it will then bypass.

EFFECT OF CRACK DEPTH RATIO ON DOUBLE-K FRACTURE PARAMETERS

The relationships of $CMOD_c$, $F_{H\max}$, a_c, K_{IC}^{ini}, and K_{IC}^{un} with the crack depth ratio α can be obtained from tests with specimens of group 1. The results are shown in **Figs. 8–12**.

The relation curves of $CMOD_c$–α, $F_{H\max}$–α, and a_c–α all can be well fitted by exponential function, whereas K_{IC}^{ini}–α can be fitted by Boltzmann function. For the K_{IC}^{un}–α, the linear

FIG. 7 Crack propagation of RCC specimen.



function is appropriate to fit the relation curve. All the fitting equations are shown in the corresponding figures above, where R is the correlation coefficient. It can be found that $CMOD_c$ and a_c increase with the increase of α, whereas $F_{H\,\max}$ decreases.

The effect of α on K_{IC}^{ini} is different from K_{IC}^{un}, α has a negative influence on K_{IC}^{ini}, i.e., K_{IC}^{ini} decreases with the increase of α. The effect of α on K_{IC}^{un} is positive, i.e., K_{IC}^{un} increases with the increase of α. In the range of $0.4 \sim 0.6$, α has a continuous effect on K_{IC}^{ini}; however, the effect on K_{IC}^{un} decreases with the increase of α.

EFFECT OF SPECIMEN SIZE ON THE UNSTABLE FRACTURE TOUGHNESS K_{IC}^{un}

The relationships of $CMOD_c$, $F_{H\,\max}$, a_c, and K_{IC}^{un} with h can be obtained from tests with specimens of group 2. The results are shown in **Figs.** 13–16, respectively.

The relation curves of $CMOD_c$–h, a_c–h can be fitted by a linear function, while $F_{H\,\max}$–h can be fitted by Gauss function. For the K_{IC}^{un}–h, the exponential function is appropriate to fit the relation curve. All the fitting curves are shown in the

FIG. 8 $CMOD_c$–α curve.

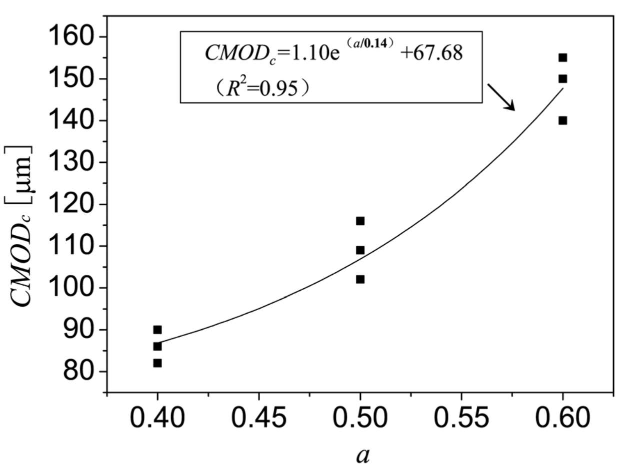


FIG. 9 $F_{H\,\max}$–α curve.

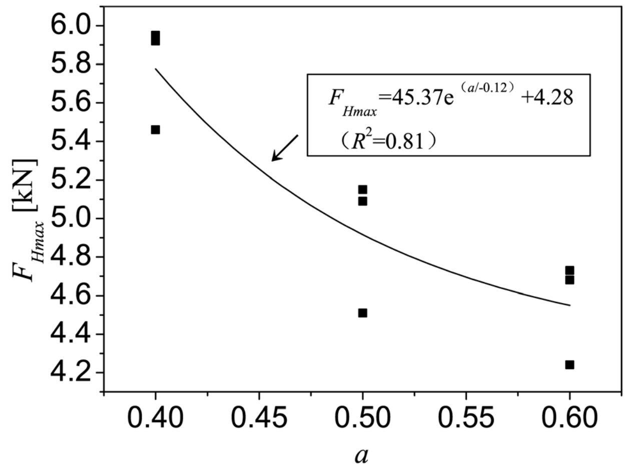


FIG. 10 a_c–α curve.

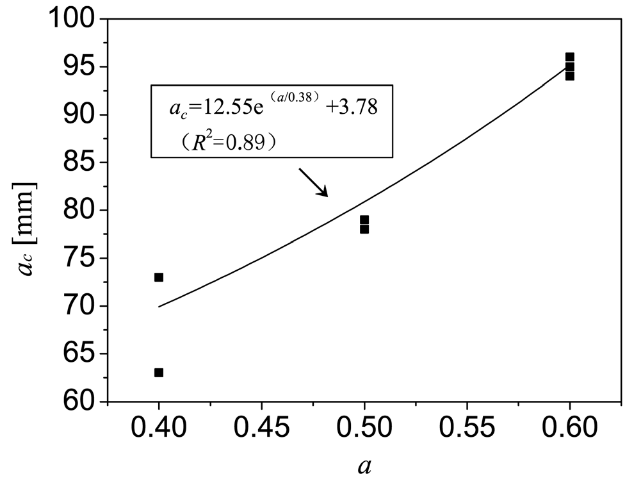


FIG. 11 K_{IC}^{ini}–α curve.

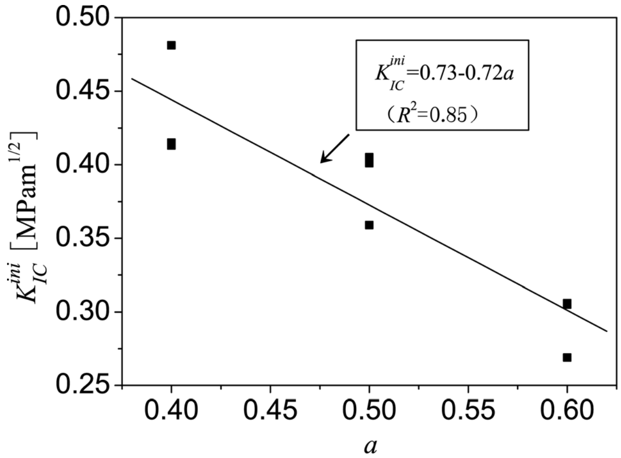


FIG. 12 K_{IC}^{un}–α curve.

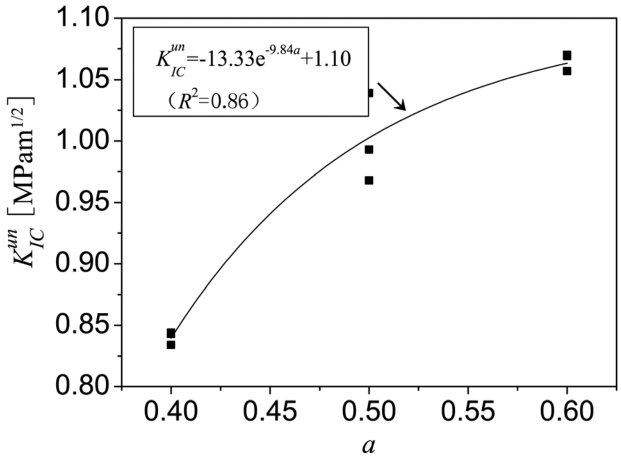


FIG. 13 $CMOD_c$–h curve.

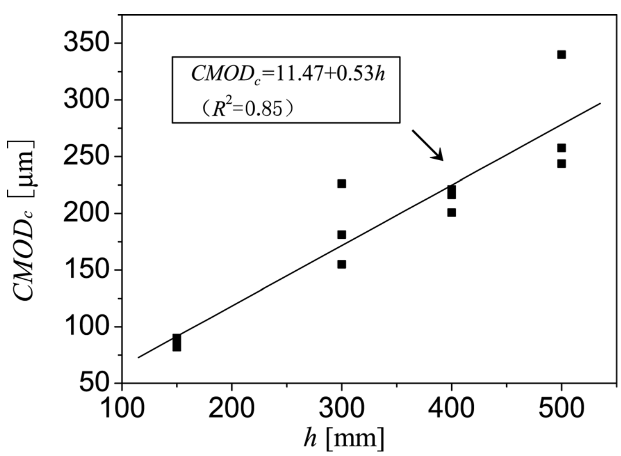


FIG. 14 $F_{H\max}$–h curve.

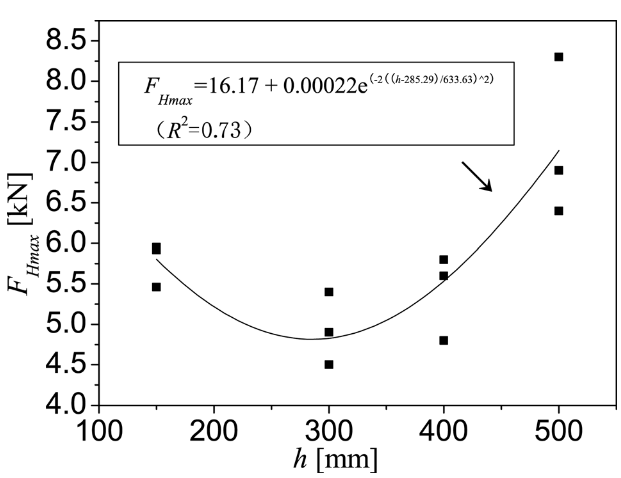
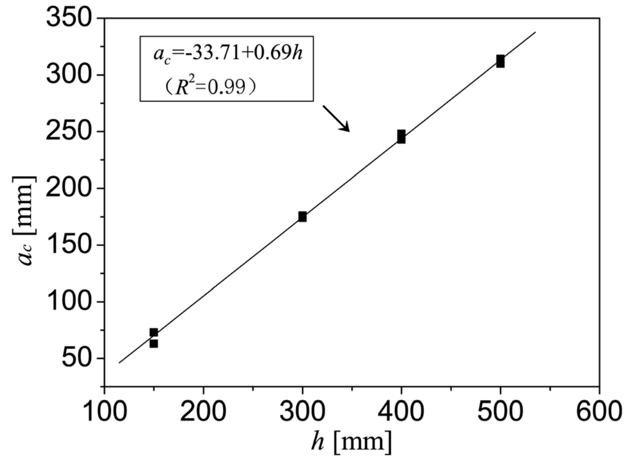


FIG. 15 a_c–h curve.



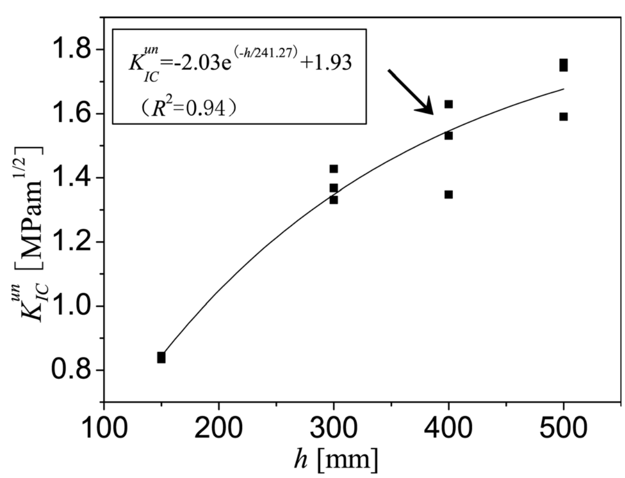
corresponding figures above. It implied that $CMOD_c$ and a_c increase with the increase of h. $F_{H\max}$ reaches the minimum when h is 300 mm.

For α is 0.4 and h is <500 mm, the results above indicated that K_{IC}^{un} increase with the increase of h. When h is <300 mm, K_{IC}^{un} grows rapidly; however, it tends to be stable when h is >300 mm. It implied that when h is <300 mm, the effect of specimen size on K_{IC}^{un} is significant, but it decreases when h is >300 mm.

Conclusions

Two groups of RCC specimens with different specimen size and crack depth ratio were investigated by wedge splitting method to study the effects of different specimen sizes and different crack depth ratios of roller compacted concrete on double-K fracture model parameters. Double-K fracture parameters were

FIG. 16 K_{IC}^{un}–h curve.



calculated by the Double-K fracture theory of concrete. The following conclusions can be drawn:

1. When the crack depth ratio α increased from 0.4 to 0.6, the unstable fracture toughness K_{IC}^{un} increases, whereas the initial fracture toughness K_{IC}^{ini} decreases. α has a continuous effect on K_{IC}^{ini}; however, the effect on K_{IC}^{un} reduces with the increase of α.
2. For α is 0.4 and h is $<$500 mm, K_{IC}^{un} increases with the increase of h. When h is $<$300 mm, K_{IC}^{un} grows rapidly; however, it tends to be stable when h is $>$300 mm. It implied that when h is $<$500 mm, the specimen size will affect K_{IC}^{un}, so if double-K fracture theory is employed as the practical crack fracture criterion, the height of the specimen should be $>$500 mm.

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